

## Converting Among Decimals, Fractions, and Percents

### I. CHANGING A FRACTION TO A DECIMAL

To change a fraction to a decimal, divide the denominator (bottom) into the numerator (top).

This works because, by definition, the fraction bar indicates division. Thus,  $\frac{4}{25}$  means "4 divided by 25."

Vocabulary:  $\text{Divisor} \overline{) \text{Dividend}}$  Quotient

#### Example I-A: Change $\frac{4}{25}$ to a decimal

1. Set up a long division problem to divide 25 into 4. -----  $25 \overline{) 4}$
2. Since 25 will not go into 4, add a decimal point and a zero -----  $25 \overline{) 4.0}$   
to the right of the 4 and bring that decimal point straight up.
3. Since 25 will go into 40 once, write 1 over the zero, then -----  $25 \overline{) 4.0}$   
multiply  $1 \times 25$  (= 25), .1  
write the result under the 4.0, (Align digits as shown) -----  $\underline{-25}$   
and subtract. ----- 15
4. Since there is a remainder, add another zero, -----  $25 \overline{) 4.00}$   
then bring that zero straight down. -----  $\underline{-25}$  .1  
150
5. Since 25 will go into 150 six times, write 6 over the last zero, then ----  $25 \overline{) 4.00}$   
multiply  $6 \times 25$  (= 150), .16  
write the result under the 150, (Align digits as shown) -----  $\underline{-150}$   
and subtract. ----- 0
6. Since the remainder is zero, we are done.  
Thus, the proper fraction  $\frac{4}{25}$  is equal to the decimal **.16**.  
*[Note the answer could also be written as **0.16**.]*

#### Example I-B: Change $\frac{13}{5}$ to a decimal

Set up a long division problem to divide 5 into 13.

Then follow steps similar to those used in **Example IA**.

*[Note that 5 goes into 13 so we do not need to add the **decimal point** and **zeroes** right away.]*

$$\begin{array}{r} 2.6 \\ 5 \overline{) 13.0} \\ \underline{-10} \phantom{0} \\ 30 \\ \underline{-30} \\ 0 \end{array}$$

Thus, the improper fraction  $\frac{13}{5}$  is equal to the decimal **2.6**.

## Converting Among Decimals, Fractions, and Percents

**Example I-C: Change  $\frac{2}{110}$  to a decimal**

1. Set up a long division problem to divide 110 into 2. -----  $110 \overline{)2}$
2. Since 110 will not go into 2, add a decimal point and a zero -----  
to the right of the 2 and bring that decimal point straight up. -----  $110 \overline{)2.0}$
3. Since 110 will not go into 20, add another zero to the right of the 0 ----  
you added in Step 2 and write a 0 in the quotient as a place holder. ----  $110 \overline{)2.00}$
4. Since 110 will go into 200 once, write 1 over the last zero, then -----  
multiply  $1 \times 110 (= 110)$ ,  $110 \overline{)2.00}$   
write the result under the 2.00, (Align digits as shown) -----  $\underline{-110}$   
and subtract. -----  $90$
5. Since there is a remainder, add another zero, -----  $110 \overline{)2.000}$   
then bring that zero straight down. -----  $\underline{-110}$   
 $900$
6. Since 110 will go into 900 eight times, write 8 over the last zero, then  
 $110 \overline{)2.000}$   
 $\underline{-110}$   
 $900$   
multiply  $8 \times 110 (= 880)$ ,  
write the result under the 900, (Align digits as shown) -----  $\underline{-880}$   
and subtract. -----  $20$

Notice that the remainder is the same as the dividend in Step 2. **Whenever the remainder is the same as a previous remainder (or dividend) we are dealing with a repeating decimal.** As shown in Step 7, we could keep adding zeroes to the dividend and see that the digits **18** repeat.

7.  $110 \overline{)2.00000}$   
 $\underline{-110}$   
 $900$   
 $\underline{-880}$   
 $200$   
 $\underline{110}$   
 $900$   
 $\underline{880}$   
 $20$

The most common way to write a repeating decimal is to draw a line over the entire repeating portion. Thus, the proper fraction  $\frac{2}{110}$  is equal to the repeating decimal **.018**.

*[Note the answer could also be written as **0.018**.]*

**PRACTICE: CHANGE EACH FRACTION BELOW TO ITS DECIMAL EQUIVALENT.**

- A.  $\frac{2}{5}$     B.  $\frac{5}{8}$     C.  $\frac{17}{5}$     D.  $\frac{2}{3}$     E.  $\frac{18}{20}$     F.  $\frac{8}{11}$     G.  $\frac{1}{32}$     H.  $\frac{2}{55}$

# Converting Among Decimals, Fractions, and Percents

## II. CHANGING A FRACTION TO A PERCENT

There are several ways to change a fraction to a percent. For now we will focus on just one method involving the use of **proportions** using the following simplified definition.

**A PROPORTION IS TWO FRACTIONS THAT ARE EQUAL.**  
 For example:  $\frac{2}{5} = \frac{40}{100}$

Furthermore, since a **percent** may be thought of as “a fraction whose denominator is 100”, our “target fraction” will always have a denominator of 100.

### Example II-A: Change $\frac{4}{25}$ to a percent.

*[Note: We will solve this first example without using algebra.]*

1. Set up a proportion with the original fraction equal to a fraction with an unknown numerator and a denominator of 100.

*[You might find it helpful to think to yourself: “4 over 25 equals ‘something’ over 100”].*

$$\frac{4}{25} = \frac{\%}{100\%}$$

2. Multiply the known values that are diagonal to each other.

$$(4 \times 100 = 400)$$

$$\frac{4}{25} \swarrow \searrow \frac{\%}{100\%}$$

3. Divide the result from Step 2 (400) ----- by the “odd man out” (25).

$$\begin{array}{r} 16 \\ 25 \overline{) 400} \\ \underline{-25} \phantom{0} \\ 150 \\ \underline{150} \\ 0 \end{array}$$

4. Check your answer by **substituting your answer for the unknown %** then **cross-multiplying** and verify you get the same answer.

*Since the answer checks, we know  $\frac{4}{25} = 16\%$ .*

*[To **cross-multiply** means to multiply the top of the first fraction by the bottom of the second and set it equal to the product of the bottom of the first fraction times the top of the second.]*

$$\frac{4}{25} \swarrow \searrow \frac{16}{100}$$

?

$$4 \times 100 = 25 \times 16$$

$$400 = 400$$

### Example II-B: Change $\frac{9}{5}$ to a percent.

*[Note: We will solve this example using an algebraic approach.]*

1. **Set up a proportion** with the original fraction equal to a fraction with a letter to hold the place of the unknown numerator and a denominator of 100.

*[You might find it helpful to think to yourself: “9 over 5 equals ‘x %’ over 100%”].*

$$\frac{9}{5} = \frac{x\%}{100\%}$$

*(Continued on next page)*

## Converting Among Decimals, Fractions, and Percents

2. **Cross-multiply.**

*[I recommend writing the product with the unknown on the left side. Also, notice in algebra we write "5x" rather than "5 × x" and "9(100)" means the same as "9 × 100".]*

$$\frac{9}{5} \times \frac{x}{100\%}$$

$$5x = 9(100)$$

3. **Simplify** by finding the product of **9 & 100**-----  $5x = 900$

4. Since we want to know the value of **x** (not **5x**), **divide both sides of the equation** by the coefficient (**5**) of the unknown term (**5x**). ----  $5x \div 5 = 900 \div 5$

5. **Simplify** by performing the indicated divisions -----  $x = 180$

6. Check your answer by **substituting your answer for the unknown %** then **cross-multiplying** and verify you get the same answer.

$$5(180) = 9(100)$$

$$900 = 900$$

*Since the answer checks, we know  $\frac{9}{5} = 180\%$ .*

### Example II-C: Change $\frac{2}{3}$ to a percent.

*[Note: As in Example IIB, we will solve this example algebraically.]*

1. **Set up the appropriate percent proportion.**

*[You might find it helpful to think to yourself: "2 over 3 equals 'x %' over 100%".]*

$$\frac{2}{3} \times \frac{x}{100\%}$$

2. **Cross-multiply.**  $3x = 2(100)$

3. **Simplify** by finding the product of **2 & 100**-----  $3x = 200$

4. **Divide both sides of the equation** by the coefficient (**3**) of the unknown term (**3x**). -----  $\frac{3x}{3} = \frac{200}{3}$   
*[Another way to indicate division is to create a fraction with the divisor as the denominator.]*

3. **Simplify** by performing the indicated divisions. (See A & B below)  
*[Method A was used to arrive at the mixed number form and Method B was used to arrive at the repeating decimal form.]*

A. 
$$\begin{array}{r} 3 \overline{) 66\frac{2}{3}} \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

B. 
$$\begin{array}{r} 3 \overline{) 66.\bar{6}} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \end{array}$$

$$x = 66\frac{2}{3}$$

or

$$x = 66.\bar{6}$$

4. **Check your answer.** -----  
*Since the answer checks, we know  $\frac{2}{3} = 66\frac{2}{3}\%$  or  $66.\bar{6}\%$ .*

$$\begin{array}{l} ? \\ 3(66\frac{2}{3}) = 2(100) \\ \frac{3}{1}(\frac{200}{3}) = 200 \\ 200 = 200 \end{array}$$

### PRACTICE: CHANGE EACH FRACTION BELOW TO ITS EQUIVALENT PERCENT.

- A.  $\frac{7}{4}$     B.  $\frac{1}{8}$     C.  $\frac{7}{15}$     D.  $\frac{5}{2}$     E.  $\frac{9}{10}$     F.  $\frac{7}{22}$     G.  $\frac{11}{32}$     H.  $\frac{22}{500}$

## Converting Among Decimals, Fractions, and Percents

### III. CHANGING A PERCENT TO A FRACTION

The word “*percent*” means “*in every 100*”. For example, if the sales tax in a certain state is six percent we know that for every \$100 spent we will pay \$6 in sales tax. We can express this tax rate as  $\frac{6}{100}$ . Thus, a percent can be viewed as a fraction with a denominator of 100.

Also, since we saw earlier that the **fraction bar** means “**divided by**”, a percent can also be viewed as a number divided by 100. Thus, we can express also this tax rate as  $6 \div 100$ .

**SUMMARY:**  $N\% = \frac{N}{100} = N \div 100$   
 For example:  $88\% = \frac{88}{100} = 88 \div 100$

#### Example III-A: Change 32% to a fraction in lowest terms.

1. Rewrite the percent as a fraction with a denominator of 100. -----  $32\% = \frac{32}{100}$

2. Fully reduce the resulting fraction by dividing the numerator and denominator by their **Greatest Common Factor (GCF)**.

- [Notes: 1. The **GCF** is the largest number that will divide evenly into both the numerator and denominator.  
 2. Dividing both the numerator and denominator by the same value is the same as dividing the original fraction by 1, which does not change its value.  
 3. Anytime the denominator is a **power of 10** (10, 100, 1000, etc.) we need only consider factors that are **products of 2's and 5's**.  
 4. Although there are fewer steps involved if we divide by the GCF, we do not always spot it. So another approach is to divide by any common factor and repeat the process until the numerator and denominator have no factors (other than 1) in common.]

$$\frac{32}{100} \div \frac{4}{4} = \frac{8}{25}$$

or

$$\frac{32}{100} \div \frac{2}{2} = \frac{16}{50}$$

$$\frac{16}{50} \div \frac{2}{2} = \frac{8}{25}$$

3. Check that you reduced correctly by **setting up a proportion that equates the original fraction to your fully reduced fraction** then **cross-multiplying** and verify you get the same answer.

$$\frac{32}{100} \begin{array}{l} \nearrow \quad \nwarrow \\ \quad \quad \quad \end{array} \frac{8}{25}$$

?

$$8 \times 100 = 32 \times 25$$

$$800 = 800$$

Thus,  $32\% = \frac{32}{100} = \frac{8}{25}$ .

#### Example III-B: Change 47.25% to a fraction in lowest terms.

1. Rewrite the percent as a fraction with a denominator of 100. ----  $47.25\% = \frac{47.25}{100}$

2. Change the numerator from decimal to a whole number by multiplying both the numerator and denominator by a **power of 10** (in this case 100) that will result in moving the decimal point to the right of the last digit of the numerator.

$$\frac{47.25}{100} \times \frac{100}{100} = \frac{4,725}{10,000}$$

## Converting Among Decimals, Fractions, and Percents

3. **Fully reduce the resulting fraction** by dividing the numerator and denominator by their **Greatest Common Factor (GCF)**.

$$\frac{4,725}{10,000} \div \frac{25}{25} = \frac{189}{400}$$

or

$$\frac{4,725}{10,000} \div \frac{5}{5} = \frac{945}{2000}$$

$$\frac{945}{2000} \div \frac{5}{5} = \frac{189}{400}$$

*[Note: Although there are fewer steps involved if we divide by the GCF, we do not always spot it. So another approach is to divide by any common factor and repeat the process until the numerator and denominator have no factors (other than 1) in common.]*

4. Check that you reduced correctly by **setting up a proportion that equates the original fraction to your fully reduced fraction** then **cross-multiplying** and verify you get the same answer.

$$\frac{47.25}{100} \quad \begin{array}{c} \nearrow \\ \searrow \end{array} \quad \frac{189}{400}$$

?

$$47.25 \times 400 = 189 \times 100$$

$$18,900 = 18,900$$

Thus,  $47.25\% = \frac{47.25}{100} = \frac{4725}{10,000} = \frac{189}{400}$ .

### Example III-C: Change $5\frac{2}{3}\%$ to a fraction in lowest terms.

1. **Rewrite the mixed number as an improper fraction** -----  $5\frac{2}{3}\% = \frac{(5 \times 3 + 2)}{3}\%$   
 $= \frac{17}{3}\%$

2. To avoid having to deal with a complex fraction, **rewrite the fractional percent using “ $\div 100$ ” in place of the % sign.** -----  $\frac{17}{3}\% = \frac{17}{3} \div 100$

3. Prepare to divide by **rewriting 100 as an improper fraction with a denominator of 1.** -----  $= \frac{17}{3} \div \frac{100}{1}$   
*[Remember, any whole number can be written as an improper fraction with a denominator of 1.]*

4. **Change the division to multiplying by the reciprocal.** -----  $= \frac{17}{3} \times \frac{1}{100}$   
*[Notes: 1. This step is justified by the **definition of division**.  
 2. Two numbers are **reciprocals** if their **product equals 1**. For example,  $\frac{3}{7}$  and  $\frac{7}{3}$  are reciprocals since  $\frac{3}{7} \times \frac{7}{3} = \frac{21}{21} = 1$ .]*

4. **Multiply straight across.** -----  $= \frac{17}{300}$

5. **Fully reduce the resulting fraction.** *[ $\frac{17}{100}$  is already fully reduced; since the only whole number, other than 1, that will divide into 17 evenly is 17 and 17 will not divide evenly into 100. Thus  $5\frac{2}{3}\% = \frac{17}{300}$ .]*

### PRACTICE: CHANGE EACH PERCENT BELOW TO ITS EQUIVALENT FULLY REDUCED FRACTION.

- |                      |           |                      |                      |
|----------------------|-----------|----------------------|----------------------|
| A. 18%               | B. 225%   | C. $33\frac{1}{3}\%$ | D. 12.125%           |
| E. $66\frac{2}{3}\%$ | F. 105.5% | G. 0.06%             | H. $62\frac{5}{8}\%$ |

## Converting Among Decimals, Fractions, and Percents

### IV. *Changing a decimal to a percent*

To change a decimal to a percent, multiply the decimal by 100. (The end result is that the decimal point moves **two places** to the **right**.)

#### Example IV-A: Change .23 to a percent

Solution: Multiply .23 by 100.

$$\begin{array}{r} .23 \\ \times 100 \\ \hline 23.00 \end{array}$$

Answer:  $.23 = 23\%$

This works because percent means “per hundred” which is the same as saying “divided by 100.” Thus, to go from decimal notation to percent notation we must “undo” the division.

#### **PRACTICE: CHANGE EACH DECIMAL BELOW TO ITS EQUIVALENT PERCENT.**

A. 0.68

B. .07

C. .0375

D. 36.8

E. 0.00125

F. 0.7

G. 150.345

H. 0.857142

## Converting Among Decimals, Fractions, and Percents

### V. CHANGING A PERCENT TO A DECIMAL

To change a percent to a decimal, divide the percent by 100. (The end result is that the decimal point moves **two places to the left**.)

#### Example V-A: Change 35% to a decimal

Solution: Divide 35 by 100.

$$\begin{array}{r} .35 \\ 100 \overline{) 35.00} \\ \underline{30 \ 0} \\ 5 \ 00 \\ \underline{5 \ 00} \\ 0 \end{array}$$

Answer:  $35\% = .35$

This works because percent means “per hundred” which is the same as saying “divided by 100.” Thus, 35% means “35 divided by 100.”

(Notice that for whole numbers the decimal point is understood to come at the end of the number.)

### PRACTICE: CHANGE EACH PERCENT BELOW TO ITS EQUIVALENT DECIMAL.

A. 23%

B. 37.6%

C.  $13\frac{2}{3}\%$

D. 5%

E. 0.087%

F. 50%

G.  $10\frac{7}{8}\%$

H.  $3.\overline{636}\%$



## Converting Among Decimals, Fractions, and Percents

### VI. CHANGING A DECIMAL TO A FRACTION

To change a decimal to a fraction: (1) re-write the number as a fraction with a denominator of 1, (2) multiply the numerator and denominator by a power of ten that will change the numerator to a whole number (use a 1 followed by the same number of zeros as there are digits to the right of the decimal point), and 3) reduce the fraction to lowest terms.

#### Example VI-A: Change 0.235 to a fraction in lowest terms

Solution: Re-write 0.235 as  $\frac{0.235}{1}$

Multiply  
Top & Bottom by 1000  $\frac{0.235}{1} \times \frac{1000}{1000} = \frac{235}{1000}$

Reduce:  $\frac{235}{1000} = \frac{47}{200}$

Answer:  $0.235 = \frac{47}{200}$

In step (1) we are dividing by 1 and in step (2) we are multiplying by 1 (since  $\frac{1000}{1000}$  means "1000 divided by 1000").

Thus, we have not changed the value of the number – only the way it looks.

As a check, divide 47 by 200 to get 0.235.

To change a decimal to a fraction: (1) use the digits of the decimal as the numerator (top) of the fraction (do not include the decimal point), (2) use a 1 followed by the same number of zeros as there are digits to the right of the decimal point as the denominator (bottom) of the fraction, and 3) reduce the fraction to lowest terms.

#### Example: Change 0.0612 to a fraction in lowest terms

Solution: Use 612 as the numerator  $\rightarrow$   $\frac{612}{10,000}$   
and 10,000 as the denominator  $\rightarrow$

Reduce:  $\frac{612}{10,000} = \frac{153}{2500}$

Answer:  $0.0612 = \frac{153}{2500}$

### **PRACTICE: CHANGE EACH DECIMAL BELOW TO ITS EQUIVALENT FRACTION IN LOWEST TERMS..**

A. 0.6

B. 2.75

C. .3

D. 10,286.008

E. .06

F. .00375

G. 52.085

H. 7.2555

## Converting Among Decimals, Fractions, and Percents

### ANSWER KEY

#### I. *Changing a fraction to a decimal*

- A. 0.4                      B. .625                      C. 3.4                      D.  $0.\overline{6}$   
E. .9                      F.  $0.\overline{72}$                       G. 0.03125                      H.  $.\overline{036}$

#### II. *Changing a fraction to a percent*

- A. 175%                      B. 12.5% or  $12\frac{1}{2}\%$                       C.  $46.\overline{6}\%$  or  $46\frac{2}{3}\%$                       D. 250%  
E. 90%                      F.  $31.8\overline{18}\%$  or  $31\frac{9}{11}\%$                       G. 34.375% or  $34\frac{3}{8}\%$                       H. 4.4% or  $4\frac{2}{5}\%$

#### III. *Changing a percent to a fraction*

- A.  $\frac{9}{50}$                       B.  $\frac{9}{4}$  or  $2\frac{1}{4}$                       C.  $\frac{1}{3}$                       D.  $\frac{97}{800}$   
E.  $\frac{2}{3}$                       F.  $\frac{211}{200}$  or  $1\frac{11}{200}$                       G.  $\frac{3}{5000}$                       H.  $\frac{501}{800}$

#### IV. *Changing a decimal to a percent*

- A. 68%                      B. 7%                      C. 3.75%                      D. 3,680%  
E. 0.125%                      F. 70%                      G. 15,034.5%                      H.  $85.7142857142\overline{\%}$

#### V. *Changing a percent to a decimal*

- A. 0.23                      B. .376                      C.  $1.33\overline{6}$                       D. .05  
E. 0.00087                      F. 0.5                      G. .10875                      H.  $0.03\overline{6}$

#### VI. *Changing a decimal to a fraction*

- A.  $\frac{3}{5}$                       B.  $\frac{11}{4}$  or  $2\frac{3}{4}$                       C.  $\frac{3}{10}$                       D.  $10,286\frac{1}{125}$   
E.  $\frac{3}{50}$                       F.  $\frac{3}{800}$                       G.  $\frac{10,417}{200}$  or  $52\frac{17}{200}$                       H.  $7\frac{511}{2000}$

For more practice ask your instructor for the **Equivalent Number Forms Practice** handout.

## Equivalent Number Forms Practice

Mixed or Whole Number	Fraction	Calculator Division	Long Division	Decimal	Percent	Ratio ("to" form)	Ratio ("colon" form)
N/A	3/4	3 ÷ 4	$\overline{4)3}$	.75 or 0.75	75%	3 to 4	3:4
1 1/4	5/4	5 ÷ 4	$\overline{4)5}$	1.25	125%	5 to 4	5:4
	7/8						
		9 ÷ 5					
			$\overline{5)2}$				
				0.75			
					262.50%		
						6 to 10	
2 4/11							
	2/3						
		9 ÷ 20					
			$\overline{20)31}$				
				3.25			
					12.5%		
						12 to 9	
							7:12
5 3/5							
	11/5						
				0.08			
					288%		
				3.006			

## Equivalent Number Forms Practice

### Conversion Practice Answer Key

Mixed or Whole Number	Fraction	Calculator Division	Long Division	Decimal	Percent	Ratio ("to" form)	Ratio ("colon" form)
N/A	3/4	3 ÷ 4	4 ) 3	.75 or 0.75	75%	3 to 4	3:4
1 1/4	5/4	5 ÷ 4	4 ) 5	1.25	125%	5 to 4	5:4
N/A	7/8	7 ÷ 8	8 ) 7	0.875	87.5%	7 to 8	7:8
1 4/5	9/5	9 ÷ 5	5 ) 9	1.8	180%	9 to 5	9:5
N/A	2/5	2 ÷ 5	5 ) 2	0.4	40%	2 to 5	2:5
N/A	3/4	3 ÷ 4	4 ) 3	0.75	75%	3 to 4	3:4
2 5/8	21/8	21 ÷ 8	8 ) 21	2.625	262.5%	21 to 8	21:8
N/A	6/10 = 3/5	6 ÷ 10	10 ) 6	0.6	60%	6 to 10	6:10 or 3:5
2 4/11	26/11	26 ÷ 11	11 ) 26	2.36	236.36%	26 to 11	26:11
N/A	2/3	2 ÷ 3	3 ) 2	0.6	66.6%	2 to 3	2:3
N/A	9/20	9 ÷ 20	20 ) 9	0.45	45%	9 to 20	9:20
1 11/20	31/20	31 ÷ 20	20 ) 31	1.55	155%	31 to 20	31:20
3 1/4	13/4	13 ÷ 4	4 ) 13	3.25	325%	13 to 4	13:4
N/A	1/8	1 ÷ 8	8 ) 1	0.125	12.5%	1 to 8	1:8
1 1/3	12/9 = 4/3	12 ÷ 9	9 ) 12	1.3	133.3%	12 to 9	12:9 or 4:3
N/A	7/12	7 ÷ 12	12 ) 7	0.583	58.3%	7 to 12	7:12
5 3/5	28/5	28 ÷ 5	5 ) 28	5.6	560%	28 to 5	28:5
2 1/5	11/5	11 ÷ 5	5 ) 11	2.2	220%	11 to 5	11:5
N/A	2/25	2 ÷ 25	25 ) 2	0.08	8%	2 to 25	2:25
2 22/25	72/25	72 ÷ 25	25 ) 72	2.88	288%	72 to 25	72:25
3 3/500	1503/500	1503 ÷ 500	500 ) 1503	3.006	300.6%	1503 to 500	1503:500